Revisiting Asymmetric Marriage Rules

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Abstract

Although generalized exchange remains an emblematical model of alliance theory, characterizing matrimonial systems as pertaining to this model is tricky. The necessary condition of generalized exchange is the deliberate preference for asymmetric exchanges. Given a marriage dataset, can we determine whether the observed pattern is due to the eralization of a social norm enjoining symmetric or asymmetric exchange or is the result of random processes? Here, relevant probabilities and indexes are established in the framework of graph theory, and are validated using a demographic individual-based model. The methods are applied to three datasets from the literature, allowing to assess with great confidence that the observed marriage configurations were not random.

Keywords: matrimonial patterns, generalized exchange, directed graph, asymmetric relation, random matrices, individual-based model

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1. Introduction

Ethnographic investigation in social anthropology proceeds by the identification of patterns that one deems characteristic of norms and practices and whose internal structure as well as links to other domains of social life are then analysed (Hammersley and Atkinson, 2007). For the ethnography of kinship, these patterns are noticeable features of kinship relationships, descent, affinity and alliance. The present article concerns the status of alliance patterns observed within genealogies and matrimonial data collected in the field. We wish to draw attention on a critical question and suggest methodological procedures to deal with it: under what conditions is it reasonable to consider that patterns of relationships displayed by a particular matrimonial system reflect intentional designs, in other words the realization of preferences by the actors? We shall consider the case of asymmetric alliance patterns, i.e. noticeable orientations within series of marriages occurring among several descent groups (clans, lineages, ...).

The conformity of practices to norms touches to important theoretical issues in the anthropology of kinship (see e.g. Fliche, 2006). Up to what point are asserted norms followed in practice? What then is the status of a norm: a model, a mental representation of the society, or a rule intended at the regulation of marriages? When patterns seemingly conforming to the norm are actually perceived in a set of practices, are they really the effect of an intentional application of the norm or are they produced randomly?

These questions also concern the interpretation of ethnographic material from other domains. Malinowski (1926, p. 120) questionned the propensity of anthropology to portray 'native law as the whole truth'. Indeed, the ethnography of kinship is particularly vulnerable to a major epistemological bias, the overvaluation of 'beautiful systems'. Ethnographers tend all the more to cherish beautiful systems when they originate from the discourse of the informants. The sophistication of kinship systems, be they terminologies or alliance structures, exerts a fascination that induces a depreciation of the contingencies of actual practices.

Modalities of alliance have been the object of an abundant literature. One of the modalities whose seductive form has attracted most attention and induced many debates is the one that Claude Lévi-Strauss called ‘generalized exchange’ and which before him was described by Dutch anthropologists as ‘asymmetrical connubium’ (Lévi-Strauss, 1949, 1969; Josselin de Jong, 1980). Based on a ‘prescription’ of marriage with explicit categories of kin (e.g. Mother’s Brother’s Daughter, MBD), generalized exchange consists in the circulation of women among descent groups according to an oriented cycle of alliances. Numerous writings have debated with sophisticated arguments mostly on the interpretation of the norm and, less often, on its realization (Needham, 1958b, 1962; Leach, 1951) (Lévi-Strauss, 1969, XVII ff.) (Parkin, 1990; Hage and Harary, 1996).

For Leach (1945, p. 68), circulating connubium has no ‘practical reality’. Needham (1957) showed that the connubium exists in Eastern Sumba but did not involve all the descent groups. Although reported as a norm in numerous Southeast-Asian societies, generalized exchange is far less documented at the level of actual marriages. The pure form, by which all marriages would follow the model, has never been found. In several cases where signs of a cyclic orientation seemed to emerge from the data, the relevance of such signs have been put into question (Leach, 1951; Ackerman, 1964). In effect, before considering the cyclicity of exchanges one should reflect on one of its necessary conditions, asymmetry, or the non-reciprocity of marriages among groups taken in pairs (using the terminology of network theory presented below, by ‘cyclicity’ we mean here a cycle of length at least 3, not a cycle of length 2 which corresponds to reciprocity). Assessing asymmetry out of a raw census of marriages is not an easy task. Obviously, no one expects to find a system that would contain exclusively asymmetrical relationships. But where should the threshold be set, beyond which a relevant sign of intentional preference for asymmetry could be confirmed?

The identification of asymmetry out of real data has been the object of a surprising debate in Volumes 66-67 of American Anthropologist (1964-65), focusing on an instance of generalized exchange among the Purum of Manipur (North-East India) reported by Das (1945) and widely commented by Needham (1958a). Among the several issues at stake, the novel one was about the definition of asymmetric alliances. Which alliances should be counted as asymmetric and how much was needed to decide that people actually preferred them? (Ackerman, 1964; Geoghegan and Kay, 1964; Needham, 1964). Although graph theory was already developing at that time (e.g. Harary, 1969), providing tools directly applicable to such problems, in the American Anthropologist debate, the arguments for or against the Purum asymmetry were built solely on the interpretations of matrices. The debate took a polemical and much confuse turn and finally some authors went as far as completely disqualifying the subject (Wilder, 1964), pretending that a matrimonial model should not be interpreted in the light of its possible realization – a position formerly adopted by Lévi-Strauss (1969, p. 193) and Leach (1945), although more carefully. This position gained momentum in the three following decades, culminating in full-fledged rejection of kinship studies by some scholars (e.g. Schneider, 1984).

We do not believe that practices can be evacuated in such a way. The logical outcome would be that in all cultural domains, discourses have no links with practices. We definitely agree that matrimonial norms pertain to other cultural domains than uniquely to the regulation of marriages, but we postulate that matrimonial choices are neither random nor determined uniquely by casual strategies. There exists regularities that can be detected using a methodical exploration of matrimonial corpuses. This standpoint seems to be increasingly assumed around the works of White, Read and the Kintip group (White, 1999; Read, 1998; Hamberger et al., 2011). A recent important contribution by Roth et al. (2013) proceeded from a question very similar to ours, about the role of chance in shaping matrimonial corpuses. It suggested to 'compare empirical alliance networks with a random baseline'. The variety of descriptors, and the sophisticated formalization of Roth et al. (2013) form a rich tool, particularly suited to the exploration of large corpuses. Here we consider in details a single feature, asymmetry, and simple methods to analyze its occurrence in the shallower corpuses collected during preliminary surveys.

We consider a social group partitioned into a number n of classes. Marriage of a girl from class i with a boy from class j creates a matrimonial relation from i to j, denoted $i \to j$. By *marriage* we mean the union of two individuals whereas by *matrimonial relation* we mean the relation between two classes i and j that is created when there is at least one marriage involving a girl from i and a boy from j. Matrimonial relations create alliances between classes. By *alliance* between class i and class j we mean that there exists either a single matrimonial relation $i \to j$ or $j \to i$, or that both relations $i \to j$ and $j \to i$ are present. In the later case, a single alliance corresponds to two matrimonial relations: in social network analysis (Wasserman and Faust, 1994), such a dyad is known as a *mutual*.

We assume exogamy: a girl cannot marry a boy from her own class. Although endogamous marriages are often recorded, we have dismissed them for simplicity, an option that does not alter our results significantly. The asymmetry rule stipulates that if the marriage of a girl from i with a boy from j has occurred (with $i \neq j$), no girl from j will be allowed to marry with a boy from i. A matrimonial relation from class i to class j is said *asymmetric* when we have $i \to j$ and not $j \to i$. It is *symmetric* when $i \to j$ has a counterpart $j \to i$ in the reverse direction. Similarly we speak of asymmetric and symmetric alliances.

In this study, we first compute the probability of a given configuration containing symmetric and asymmetric matrimonial relations, assuming that matrimonial relations occur at random. A statistical test allows to assess whether the degree of asymmetry of an observed configuration should be attributed to chance. The applicability of the formula to real data is validated by a demographic individual-based model. We also consider asymmetry in the set of individual marriages and define an asymmetry index for this set. The relevance of this index is tested using the demographic model together with the generation of random marriage matrices.

Asymmetry in matrimonial relations and in the set of individual marriages are two distinct notions. Asymmetry in marriages necessitates the knowledge of the number of marriages between classes whereas asymmetry in matrimonial relations can be based on more fuzzy information, e.g. 'girls from class i tend to marry with boys from class j whereas girls from class j tend not to marry with boys from class i'. Nevertheless, when the number of marriages is known, it provides information about the existence and direction of matrimonial relations, e.g. many more marriages from i to j than from j to i suggests the asymmetric matrimonial relation $i \to j$.

Our methods are applied to three observed marriage datasets from the literature, allowing to assess if these configurations should be attributed to a random process or to the deliberate application of a social norm.

2. Combinatorial study

The situation is usually described (e.g. Hamberger et al., 2011) by a directed graph with n vertices labeled $1, 2, \ldots, n$ representing the classes. An arc $i \to j$ joining vertex i to vertex j represents a matrimonial relation from class i to class j. The exogamy rule means that the directed graph does not have loops.

A directed graph with n vertices is conveniently described by its adjacency matrix, a $(0,1)$-matrix $\mathbf{A} = (A_{ij})$ of size $n \times n$ such that entry (i, j) is 1 when there is an arc from vertex i to vertex j and 0 otherwise. By the exogamy rule, the diagonal entries of $\mathbf{A}$ are 0.

To study asymmetry in the set of individual marriages, we use a weighted directed graph having the same vertices: there exists an arc $i \to j$ only when the number W_{ij} of marriages of girls from class i with boys from class j is nonzero. The integer $W_{ij} > 0$ is then associated with the arc. By the exogamy rule, $W_{ii} = 0$.

2.1. The probability of a given configuration of matrimonial relations

Let us assume that there are k matrimonial relations, among which a are asymmetric. The adjacency matrix $\mathbf{A} = (A_{ij})$ has k nonzero entries. Asymmetry of the relation $i \to j$ means that if $A_{ij} = 1$ $(i \neq j)$ then $A_{ji} = 0$. Symmetry means that both $A_{ij} = 1$ and $A_{ji} = 1$. The non-diagonal entry pairs (A_{ij}, A_{ji}) of the adjacency matrix are in number $\frac{n(n-1)}{2}$. They are of the form $(0,0)$ (no relation), $(0,1)$ or $(1,0)$ (asymmetric relation), or $(1,1)$ (pair of symmetric relations, i.e., symmetric alliance). The $k-a$ symmetric relations come in pairs. Thus $k-a$ is even so that we write $k = a+2s$. We note that $a+s$ is the number of alliances (of which a are asymmetric and s are symmetric).

The number of configurations on n classes involving exactly k matrimonial relations, of which a are asymmetric is

$$M_{\text{config}}(n, k = a + 2s) = \binom{\frac{n(n-1)}{2}}{a+s}\binom{a+s}{a}2^a.$$

Here $\binom{\frac{n(n-1)}{2}}{a+s}$ and $\binom{a+s}{a}$ are binomial coefficients. Indeed, we first choose $a+s$ of the $\frac{n(n-1)}{2}$ non-diagonal entry pairs of the adjacency matrix to be nonzero. Then we choose a of these pairs to be of one of the 2 asymmetric forms $(0,1)$ or $(1,0)$. The remaining pairs are the symmetric pairs of the form $(1,1)$.

The total number $M(n,k)$ of configurations on n classes involving exactly k matrimonial relations between classes is

$$M(n,k) = \binom{n(n-1)}{k}.$$

Indeed, k of the $n(n-1)$ non-diagonal entries of the adjacency matrix are chosen to be nonzero. The remaining entries are 0.

We obtain the probability to have a configuration of k exogamic random matrimonial relations between n classes, among which a are asymmetric:

$$p_{\text{config}}(n, k = a + 2s) = \frac{\binom{\frac{n(n-1)}{2}}{a+s}\binom{a+s}{a}2^a}{\binom{n(n-1)}{k}}. \tag{1}$$

In particular, the probability to have a configuration of k exogamic random matrimonial relations between n classes that is totally asymmetric $(s = 0)$ is

$$p_{\text{asym}}(n,k) = \frac{\binom{\frac{n(n-1)}{2}}{k}2^k}{\binom{n(n-1)}{k}}. \tag{2}$$

The formula for the number of configurations M_{config} above appears in Wasserman and Faust (1994, p. 548) with reference to the work of Holland

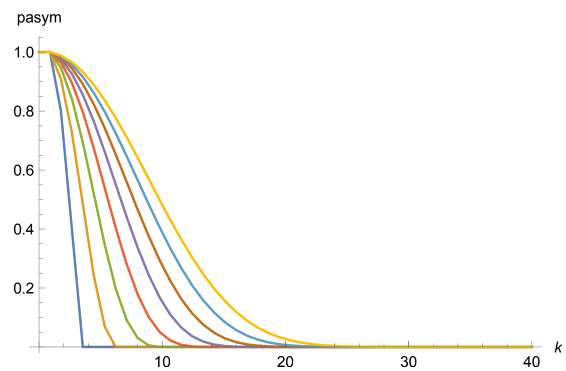

Figure 1: Probability p_{asym} to have k asymmetric matrimonial relations between n classes for $n = 3$ to $n = 10$ classes (blue to yellow).

and Leinhardt (1976). The related approach of Katz and Powell (1955) concerning mutual choice in social networks has limitations (Mandel, 2000).

We observe that for a small number k of matrimonial relations – a realistic situation in a small village where only a limited number of marriages takes place within a time interval, the probability to get an asymmetric configuration is high (Fig. 1). This is expected because when the number of random matrimonial relations is small, the probability of a symmetric pair of relations is small, so that the probability of total asymmetry is large. We also see that, for fixed k, the probability $p_{\text{asym}}(n, k)$ increases with the number n of classes.

Given an observed configuration of matrimonial relations having k_{obs} arcs of which a_{obs} are asymmetric, we wish to quantify the confidence that the observed configuration is not random. To this end, the probability distribution $p(a) = p_{\text{config}}(n, k, a)$ parameterized by the number a of asymmetric relations, is built using Eq. 1 (Fig. 5). From the expectation $E(a) = \sum_a ap(a)$ and standard deviation $\sigma(a)$ of the distribution, the z-score

$$Z(a_{\text{obs}}) = \frac{a_{\text{obs}} - E(a)}{\sigma(a)} \tag{3}$$

is computed. Assuming that the a's are normally distributed, and denoting $z = |Z|$ the absolute value of the z-score, the quantity

$$\alpha = 2\Phi(z) - 1$$

is calculated. Here Φ is the cumulative distribution of the normal distribution. Then the confidence to reject the random model is $100 \times \alpha$ in percentage.

2.2. The probability of asymmetric matrimonial patterns

The previous section considers the number of configurations of matrimonial relations, specifying the names of the classes (the labels assigned to the vertices of the directed graph). If we disregard the labels, we look for matrimonial patterns. For example, the two configurations $i \to j \to k \to i$ and $j \to i \to k \to j$ correspond to a unique cyclic pattern between 3 classes.

The number $P_{\text{asym}}(n,k)$ of asymmetric matrimonial patterns on n classes and involving exactly k matrimonial relations, $k = 0, \ldots, \frac{n(n-1)}{2}$ can be computed explicitly, but the formula is rather complicated (Davis, 1953; Harary, 1957) (Table 1). The number $P(n,k)$ of exogamic matrimonial patterns on n classes and involving exactly k matrimonial relations, $k = 0, \ldots, n(n-1)$, is the number of unlabeled directed graphs on n vertices without loops having k arcs (Harary, 1969, pp. 226-230), sequence A052283 in the On–Line Encyclopedia of Integer Sequences (Sloane, 2010) (Table 2).

Table 1: $P_{\text{asym}}(n,k)$: number of asymmetric matrimonial patterns on n classes and involving k matrimonial relations, $k = 0, \ldots, \frac{n(n-1)}{2}$.

$n \setminus k$	0	1	2	3	4	5	6	7	8	9	10	
1	1											1
2	1	1										2
3	1	1	3	2								7
4	1	1	4	10	12	10	4					42
5	1	1	4	13	41	78	131	144	107	50	12	582

Table 2: $P(n,k)$: number of exogamic matrimonial patterns on n classes and involving k matrimonial relations, $k = 0, \ldots, n(n-1)$. Note that $P(n,k) = P(n, n-k)$.

$n \setminus k$	0	1	2	3	4	5	6	7	8	9	10	
1	1											1
2	1	1	1									3
3	1	1	4	4	4	1	1					16
4	1	1	5	13	27	38	48	38	27	13		218
5	1	1	5	16	61	154	379	707	1155	1490	1670	9608

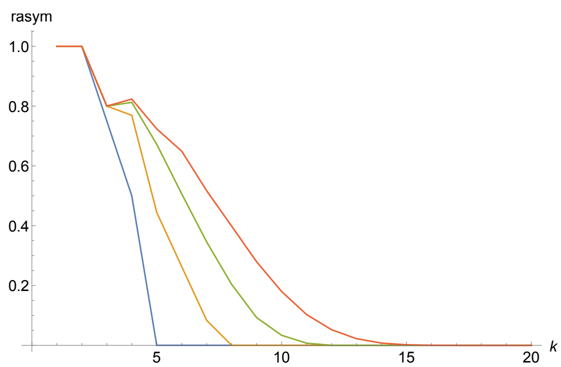

Figure 2: Probability r_{asym} to have an asymmetric matrimonial pattern involving 3 to 6 classes (blue to red) as a function of the number k of matrimonial relations.

The probability to have a totally asymmetric matrimonial pattern between n classes involving k exogamic matrimonial relations is

$$r_{\mathrm{asym}}(n,k) = \frac{P_{\mathrm{asym}}(n,k)}{P(n,k)}. \tag{4}$$

As for the matrimonial configurations, it is found that the probability to have an asymmetric matrimonial pattern is high for a small number k of matrimonial relations (Fig. 2). Also, for fixed k, the probability $r_{\mathrm{asym}}(n,k)$ increases with the number n of classes.

2.3. Asymmetry index of individual marriages

In the previous sections, we were interested in the asymmetry of configurations and patterns across classes within a society. We now address the question of asymmetry in the set of individual marriages between classes. The number of individual marriages is recorded in a matrix $\mathbf{W} = (W_{ij})$. The total number of marriages is

$$N = \sum_{i,j} W_{ij}.$$

Let us consider the marriages $i \to j$, in number W_{ij}, and the marriages $j \to i$, in number W_{ji}. Then the absolute value of the difference $W_{ij} - W_{ji}$ counts the number of marriages involving classes i and j that do not have a symmetric counterpart. The number of marriages contributing to asymmetry of the whole set of marriages is thus

$$N_{\text{asym}} = \sum_{(i,j)\in U} |W_{ij} - W_{ji}|,$$

where the sum is performed over the upper triangular part of the matrix. When there are N marriages of which N_{asym} have no symmetric counterpart, we define the asymmetry index by

$$q_{\text{asym}} = \frac{N_{\text{asym}}}{N}. \tag{5}$$

Once the asymmetry index q_{obs} of an observed marriage matrix comprising N marriages has been obtained, we want to estimate whether this index is the result of random marriages (the random model) or not. To this end, a large set of random marriage matrices of size $n \times n$ with N nonzero entries is generated (Appendix A). The mean $\bar{q}$ and standard deviation $s(q)$ of the asymmetry indexes of these matrices provide the z-score of the observed index,

$$Z(q_{\text{obs}}) = \frac{q_{\text{obs}} - \bar{q}}{s(q)}, \tag{6}$$

allowing to quantify the confidence to reject the random model.

The asymmetry index q_{asym} (Eq. 5) has been considered by Squartini et al. (2013) in a slightly different form: their measure of reciprocity r can be shown to verify $q_{\text{asym}} = 1 - r$. Another asymmetry index has been proposed by Roth et al. (2013): in our framework it is written

$$qR_{\text{asym}} = 1 - \frac{\sum_{i,j} W_{ij} W_{ji}}{\sum_{i,j} {W_{ij}}^2}, \tag{7}$$

and leads to results similar to those of index q_{asym} above.

2.4. Asymmetry index of matrimonial relations

Assuming that the number of marriages between classes is known, we can reconstruct hypothetical matrimonial relations. One way to do that is to build the **A**-matrix by replacing each nonzero entry of the **W**-matrix by 1. This option will be reexamined in the examples below (section 4).

The formulas for the asymmetry index of the integer valued matrix $\mathbf{W}$ presented above can be applied to the binary matrix $\mathbf{A}$ that has the same nonzero entries. It appears that both indexes (5) and (7) lead to the same binary asymmetry index

$$b_{\text{asym}} = \frac{a}{k},$$

the proportion of asymmetric matrimonial relations.

As for the case of marriages, a large set of random matrimonial relation matrices of size $n \times n$ with k nonzero entries can be generated (Appendix A). The mean $\bar{b}$ and standard deviation $s(b)$ of the binary asymmetry indexes of these matrices provide the z-score of the observed binary index,

$$Z(b_{\text{obs}}) = \frac{b_{\text{obs}} - \bar{b}}{s(b)}. \tag{8}$$

This approach is in fact intrisically the same as the exact formulation of section 2.1: (8) approximates (3) very well.

3. Demographic model

To validate the combinatorial study, we use a realistic individual-based population dynamics model in discrete time (the time step is of one year) that simulates the population trajectories of a small village in a rural area. A detailed description of the demographic model, built from an age-classified life cycle with observed demographic parameters (Caswell, 2001; United Nations, 1982), is given in Appendix B.

The population is regulated (density dependence), so that population size is roughly constant over time, at the carrying capacity K (the maximum number of individuals the environment can sustain). The population is partitioned into n classes and exogamic marriages are drawn at random between classes under age and cultural constraints: for each non married girl that could get married according to age and cultural rules a list of non married boys is determined; if the list of boys is non empty, a random element is drawn that becomes the husband of the girl.

At initialization, a pool of K individuals is created. The initial individuals, males and females in equal proportions and of the same age (20 years), are partionned into the n classes at random. To these males and females, the exogamic marriage procedure described above is applied every year, and newborns progressively enter the population, inheriting the class of their father. The uneven initial population structure creates transient oscillatory dynamics that are stabilized by time $t = 100$. For that reason the quantities of interest are computed from time 100.

3.1. Probability of asymmetric configuration of matrimonial relations

The matrix $\mathbf{A} = (A_{ij})$ where $A_{ij} = 1$ if and only if there occurs a marriage of a i-girl with a j-boy and $A_{ij} = 0$ otherwise, records the matrimonial relations between classes along time. A novel matrix is used when the number k of nonzero entries exceeds $n(n-1)/2$. For each value of k, the distinct matrices that appear in the simulation are stored in a list. The current matrix is compared to those already stored and tested for total asymmetry. If not already stored, the matrix is appended to the list. In this way, all distinct configurations appearing in the simulation are recorded: for each number k of entries of $\mathbf{A}$, the number of configurations $\widetilde{M}(n, k)$ involving k matrimonial relations and the number $\widetilde{M}_{\text{asym}}(n, k)$ of those configurations that are asymmetric are obtained. The probability that a configuration of k matrimonial relations between n classes is asymmetric is now estimated by

$$\widetilde{p}_{\text{asym}}(n, k) = \frac{\widetilde{M}_{\text{asym}}(n, k)}{\widetilde{M}(n, k)}.$$

The simulation shows that the theoretical result (Eq. 2) is well preserved when marriages take place under realistic conditions (Fig. 3). It should be noted that the program is not designed to reproduce formula (2) by stochastic means, but shows that the theoretical result holds within a realistic demographic setting.

3.2. Asymmetry in individual marriages

The marriage matrix $\mathbf{W}$ is updated along time, recording the marriages between classes. A novel matrix is used when the number N of marriages

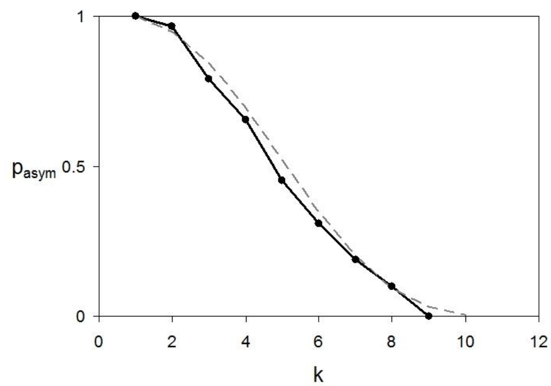

Figure 3: Probability to have k asymmetric matrimonial relations between 5 classes: theoretical values (dotted) and values obtained for a typical trajectory of the demographic model (carrying capacity $K = 1000$, 100 years).

exceeds $n(n-1)$. For each value of N, the index q_{asym} of $\mathbf{W}$ is computed according to Eq. 5.

Figure 4 displays the asymmetry index q_{asym} as a function of N for a typical population trajectory. The index appears to be high for small values of N. It is compared to the average asymmetry index of a set of random marriage matrices (thin line in Fig. 4), showing once again that demography does not bias the results.

4. Application of the methods

We apply our theoretical results to three cases: two cases assumed to represent asymmetric systems, and one a symmetric system (summary in Table 6). In all cases, the marriage matrix $\mathbf{W}$ is known. The matrimonial relation matrix $\mathbf{A}$ is constructed from $\mathbf{W}$ according to two models:

- *Model 1 - theoretical.* The network of matrimonial relations has exactly the same arcs as the weighted marriage network (see section 2.4).

- *Model 2 - realistic.* A matrimonial relation $i \to j$ is asymmetric when the number of marriages from i to j is at least twice the number of (nonzero)

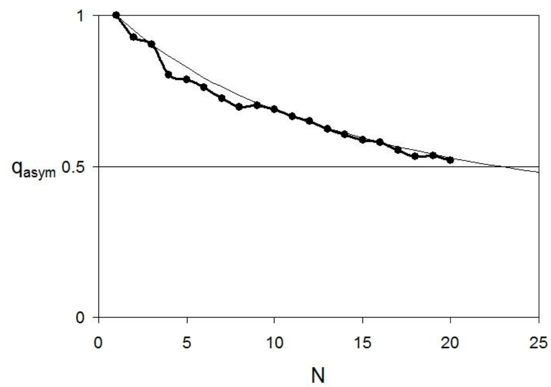

Figure 4: Index of asymmetry of N random marriages between 5 classes for a typical trajectory of the demographic model (carrying capacity $K = 1000$, 100 years). The thin curve is obtained by generating 100000 random marriage matrices and averaging their asymmetry indexes, independently of demography.

marriages from j to i, and symmetric otherwise.

In Model 1, any nonzero number of marriages of girls from class i with boys from class j determines a matrimonial relation $i \rightarrow j$. The more realistic Model 2 accounts for the fact that when the 'flows' of marriages $i \rightarrow j$ and $j \rightarrow i$ are dissimilar the corresponding matrimonial relation is likely to be asymmetric, and when the flows are similar the relation is likely to be symmetric. For example, the dyad of marriages $(1, 5)$ leads to the symmetric dyad of matrimonial relations $(1, 1)$ in Model 1, and to the asymmetric dyad $(0, 1)$ in Model 2 (no matrimonial relation in the reverse direction): informants are unlikely to judge a marriage relation symmetric only because a single marriage in the 'false' direction occurs.

4.1. Purum

Purum village in Manipur (North-East India) is one of the paradigmatic cases of generalized exchange (Das, 1945; Needham, 1958a). Its features have been the object of much debate (see the Introduction). Our marriage matrix (Table 3) is drawn from the primary source, Das (1945). There are $n = 13$ classes consisting of patrilineal descent groups and corresponding to 'sibs' in

Das' terminology. It is at the level of these sibs that a rule of asymmetry was prescribed among the Purum (Das, 1945, p. 123).

Matrimonial relations - Model 1. There are $k = 28 + 2 \times 6 = 40$ matrimonial relations, of which $a_{\text{Purum}} = 28$ are asymmetric. The probability of this observed configuration to be random is $p_{\text{Purum}} = p_{\text{config}}(13, 40, 28) = 0.1930$ by Eq. 1. Figure 5 displays the distribution of $p_{\text{config}}(13, k = 40 = a + 2s)$ when s varies (so that $a = k - 2s$). Using Eq. 3, the z-score is $Z(a_{\text{Purum}}) = -0.58$, indicating that the configuration would be random with a confidence of 57%. The random model is not rejected.

Matrimonial relations - Model 2. There are $k = 32 + 2 \times 2 = 36$ matrimonial relations, of which $a_{\text{Purum}} = 32$ are asymmetric. Using Eq. 3, the z-score is $Z(a_{\text{Purum}}) = 1.32$, indicating that the configuration is asymmetric with a confidence of 81% to reject the random model.

Marriages. The number of marriages is $N = 141$, of which $N_{\text{asym}} = 121$ have no symmetric counterpart, leading to the asymmetry index $q_{\text{Purum}} = 0.8582$ (Eq. 5). The observed index is compared to the average index of 100000 random marriage matrices. Figure 6 shows that q_{Purum} has a very high probability of not being the result of random marriages. The z-score (Eq. 6) is $Z(q_{\text{Purum}}) = 6.24$, implying that asymmetry of the set of marriages is nonrandom with 99.9999999% confidence.

4.2. Wailolong

Wailolong village in Flores island (East Indonesia) represents another rare case where a series of consistent data, initially collected by Kennedy (1955), can be used to estimate how far a prescription for asymmetry translates into actual practice (Graham, 1964; Barnes, 1977; Josselin de Jong, 1980). As stated by Graham (1964, p. 41), the interpretation of Kennedy's raw notes requires caution; we rely on Barne's understanding of Kennedy's data. Classes here correspond to patrilineages. Although the clans containing these lineages may exchange women in both directions, this was explicitly prohibited among lineages (Barnes, 1977, p. 142). The marriage matrix (Table 4) contains three

Table 3: Marriage matrix of Purum village compiled from Das (1945, Tables X and XI). Girls are in columns, as in Ackerman (1964), contrarily to the convention of the text where girls are in rows. This does not affect the results.

	M1	M2	M3	M4	MK1	MK2	K1	K2	T1	T2	T3	T4	P
M1	0	0	0	0	4	0	3	1	0	0	0	0	0
M2	0	0	0	0	0	0	0	0	2	4	0	0	0
M3	0	0	0	0	0	0	0	0	4	0	0	0	0
M4	0	0	0	0	1	0	2	1	0	0	0	0	0
MK1	0	5	0	0	0	0	18	3	0	0	0	0	2
MK2	0	0	0	0	0	0	2	0	0	0	0	0	0
K1	0	2	0	2	0	1	0	3	3	1	0	4	10
K2	0	2	0	0	2	2	0	0	4	0	0	1	1
T1	0	0	0	0	2	0	0	0	0	0	0	0	6
T2	0	0	0	0	10	2	0	0	0	0	0	0	4
T3	0	0	0	0	0	0	0	0	0	0	0	0	0
T4	0	0	0	0	2	0	1	0	0	0	0	0	0
P	4	4	0	10	0	0	3	3	0	0	0	0	0

endogamic relations from four endogamic marriages. These marriages are removed for the present study, giving $n = 17$ classes.

Matrimonial relations - Model 1. There are $k = 41 + 2 \times 5 = 51$ matrimonial relations, of which $a_{\text{Wailolong}} = 41$ are asymmetric. The probability of this configuration under random relations is $p_{\text{Wailolong}} = p_{\text{config}}(17, 51, 41) = 0.2151$. The configuration is estimated random with confidence 87% (z-score $Z(a_{\text{Wailolong}}) = -0.17$).

Matrimonial relations - Model 2. There are $k = 42 + 2 \times 4 = 50$ matrimonial relations, of which $a_{\text{Wailolong}} = 42$ are asymmetric. The configuration is estimated random with confidence 77% (z-score $Z(a_{\text{Wailolong}}) = 0.30$).

Marriages. The number of (exogamic) marriages is $N = 78$, of which $N_{\text{asym}} = 68$ have no symmetric counterpart. The asymmetry index is $q_{\text{Wailolong}} = 0.8718$. The z-score is $Z(q_{\text{Wailolong}}) = 1.52$ allowing to assess with 87% confidence that asymmetry of the set of marriages is nonrandom.

4.3. Terku Vandanam

In the case of Terku Vandanam village in Tamil Nadu (India), populated by the Kondaiyankottai Maravar caste (Good, 1981), asymmetry is not expected

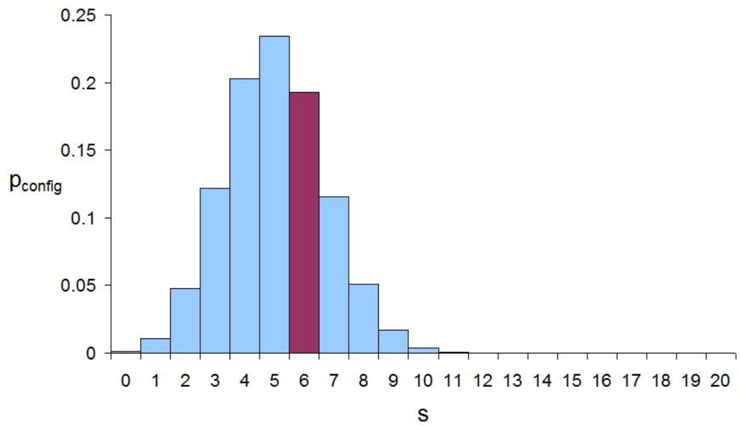

Figure 5: Distribution of $p_{\text{congig}}(n, k = a + 2s)$ for Purum village as a function of the number s of symmetric alliances, under the assumption that $k = 40$ matrimonial relations between the $n = 13$ classes are random. The red bar corresponds to p_{Purum} with $a = 28$ asymmetric alliances and $s = 6$ symmetric alliances (Model 1).

Table 4: Marriage matrix of Wailolong village (Barnes (1977), compiled from Kennedy (1955)). The 4 endogamic marriages (one within Ib, one within IIIa2, and 2 within IIIc) are removed for the study of asymmetry.

	Ia	Ib	Ic1	Ic2	IIa	IIb	IIc	IIIa1	IIIa2	IIIb	IIIc	IVa1	IVa2	IVb	IVc	Va	Vb
Ia	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
Ib	0	1	0	0	1	0	0	1	0	3	0	0	0	0	0	0	0
Ic1	0	0	0	0	0	0	0	1	0	5	2	0	0	0	0	0	0
Ic2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
IIa	1	3	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0
IIb	0	0	1	0	0	0	0	4	1	2	0	0	0	0	0	0	0
IIc	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0
IIIa1	0	1	1	1	0	0	0	0	0	0	0	1	0	0	0	1	2
IIIa2	1	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0
IIIb	0	0	0	0	4	0	0	0	0	0	0	0	1	0	0	0	3
IIIc	0	0	0	0	4	1	0	1	0	0	2	0	0	0	3	0	1
IVa1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
IVa2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
IVb	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
IVc	1	0	1	0	0	2	1	0	0	0	0	0	0	0	0	0	0
Va	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
Vb	1	0	0	0	2	2	1	0	0	0	0	0	0	0	0	0	0

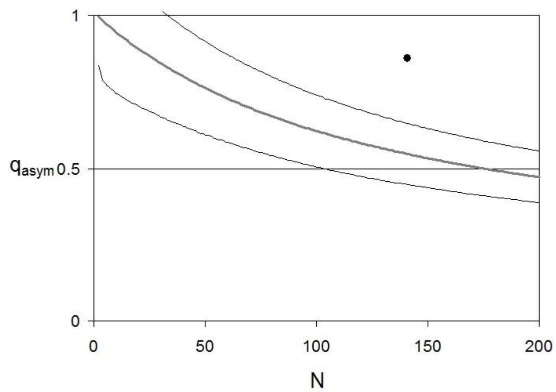

Figure 6: Purum village. The thick curve displays the average value $\bar{q}$ of the asymmetry index of N random marriages between $n = 13$ classes, obtained by generating random marriage matrices as in Fig. 4. The thin curves above and below the thick curve correspond to $\bar{q} \pm 2s(q)$ with $s(q)$ the standard deviation. The dot corresponds to the index of asymmetry q_{Purum} of the Purum marriages in number $N = 141$, well outside the 2σ confidence interval.

but considered in order to test our methods. Contrary to the Purum and Wailolong, the Kondaiyankottai Maravar do not prescribe asymmetric marriages but rather marriages with any partner falling in the cross-cousin terminological category (on the Dravidian kinship terminology, see Dumont (1975)). The $n = 7$ matrimonial classes used here are matrilineages.

Matrimonial relations - Model 1. There are $k = 6+2\times 10 = 26$ matrimonial relations of which $a_{\mathrm{Terku}} = 6$ are asymmetric. The probability of this configuration under random relations is $p_{\mathrm{Terku}} = p_{\mathrm{config}}(7, 26, 6) = 0.0626$. The configuration is symmetric, nonrandom with 94% confidence (z-score $Z(a_{\mathrm{Terku}}) = -1.90$).

Matrimonial relations - Model 2. There are $k = 9 + 2 \times 7 = 23$ matrimonial relations of which $a_{\mathrm{Terku}} = 9$ are asymmetric. The configuration is symmetric, nonrandom with 52% confidence (z-score $Z(a_{\mathrm{Terku}}) = -0.72$). In this case, the confidence of Model 2 is lower than that of Model 1 because Model 2 tends to reinforce asymmetry.

Marriages. The number of marriages is $N = 119$, of which $N_{\mathrm{asym}} = 27$ have no symmetric counterpart, leading to the asymmetry index $q_{\mathrm{Terku}} = 0.2269$. The marriage configuration is symmetric nonrandom with 93% confidence (z-score $Z(q_{\text{Terku}}) = -1.84$).

Table 5: Marriage matrix of Terku Vandanam village (Good, 1981, p. 120)

	A	B	C	D	E	F	G
A	0	16	8	2	10	1	0
B	12	0	7	6	1	0	0
C	8	1	0	1	6	0	1
D	3	3	1	0	0	0	1
E	7	2	7	1	0	5	0
F	0	0	0	0	7	0	0
G	0	0	0	0	1	1	0

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5. Discussion

We have explored whether asymmetric or symmetric configurations in a population partitioned into classes could be attributed to chance realization of marriages or matrimonial relations. These configurations concern three levels of increasing generality:

1. Individual level - marriage configuration ('Marriage of girl G from class i with boy B from class j does not have a symmetric counterpart': probability q_{asym}, Eq. 5),
2. Class level - configuration of matrimonial relations ('Matrimonial relation between class i and class j is asymmetric' : probability p_{asym}, Eq. 2),
3. Society level - matrimonial pattern ('There is an overall asymmetric pattern in matrimonial relations': probability r_{asym}, Eq. 4).

When a norm conditions the matrimonial practices in a social group, real life constraints (e.g. the absence of a partner in the preferred class because of demographic stochasticity) influence the realization of the norm: the intended rule cannot always be applied, up to the point where the norm becomes difficult to perceive in observed data. For example, in the case of Purum village, the normative matrices of matrimonial relations given by Ackerman (1964) and Needham (1962, Table 6, p. 80) lead to very low probabilities that their asymmetry would

Table 6: Descriptors of the three datasets studied.

	Purum	**Wailolong**	**Terku Vandanam**
n nb of classes	13	17	7
Matrimonial relations - Model 1			
k nb of matrimonial relations	40	51	26
a nb of asymmetric relations	28	41	6
s nb of symmetric alliances	6	5	10
a/k binary asymmetry index	0.7	0.80	0.23
$p_{\text{config}}(n, k = a + 2s)$	0.1930	0.2151	0.0626
$E(a)$	29.94	41.59	10.15
$\sigma(a)$	3.365	3.543	2.182
z-score	-0.58	-0.17	-1.90
Conclusion	RANDOM	RANDOM	SYM
Confidence	56%	87%	94%
Matrimonial relations - Model 2			
k nb of matrimonial relations	36	50	23
a nb of asymmetric relations	32	42	9
s nb of symmetric alliances	2	4	7
a/k binary asymmetry index	0.89	0.84	0.39
$p_{\text{config}}(n, k = a + 2s)$	0.1158	0.2229	0.2668
$E(a)$	27.87	40.96	10.66
$\sigma(a)$	3.129	3.488	2.298
z-score	1.32	0.30	-0.72
Conclusion	ASYM	RANDOM	SYM
Confidence	81%	77%	52%
Individual marriages			
N nb of marriages	141	78	119
N_{asym}	121	68	27
$q = N_{\text{asym}}/N$ asymmetry index	0.8582	0.8718	0.2269
qR asymmetry index	0.8938	0.9176	0.0867
$\bar{q}$	0.5462	0.7807	0.3282
$s(q)$	0.0500	0.0600	0.0550
z-score	6.24	1.52	-1.84
Conclusion	ASYM	ASYM	SYM
Confidence	99%	87%	93%

be generated at random (respectively: $p_{\text{config}}(13, 49, 0) = 1.1 \times 10^{-5}$, nonrandom with 99.9999% confidence; $p_{\text{config}}(13, 54, 1) = 3 \times 10^{-6}$, nonrandom with 99.99999% confidence). But, has we have seen, the asymmetry of the realized relations was less marked (Table 6).

In a different direction, we have observed that for a low number k of matrimonial relations between n classes, there is a high probability that random relations produce an asymmetric configuration (Fig. 1) or an asymmetric pattern (Fig. 2): a seemingly deliberate pattern emerges from randomness. A similar phenomenon has been described by Moreno and Jennings (1938).

As underlined, asymmetry in configurations of matrimonial relations and in the set of individual marriages are two distinct notions, interpreted above as occuring at distinct levels of the society (1 and 2). These notions were studied using respectively a binary matrix $\mathbf{A}$ and an integer valued matrix $\mathbf{W}$. In the examples of section 4, the number of marriages was known; matrix $\mathbf{A}$ was deduced from matrix $\mathbf{W}$ using two models, a theoretical one (Model 1), and a more realistic one (Model 2). Overall, asymmetry indexes based on marriages appeared more efficient than those based on matrimonial relations (Table 6). This is expected in this case, because $\mathbf{W}$ contains more information than $\mathbf{A}$. Nevertheless, in situations where marriage counts are not known, $\mathbf{W}$ can be constructed from ethnographic data: the methods developped in section 2.1 allow to assess the presence of a nonrandom symmetric or asymmetric configuration of matrimonial relations, with a confidence index that quantifies the degree to which the social norm has been followed. To account for potential biases associated with the quality or reliability of the informants, several scenarios can be built with uncertain relations $i \to j$ either included or withdrawn.

In the examples, even if the configuration of individual marriages could be assessed to be nonrandom with high confidence, by comparing the observed asymmetry index to the asymmetry index of random matrices (Fig. 6), the configuration of matrimonial relations deduced from the marriage data (Model 1 and Model 2) could appear to have been generated by a random process. This is the case of Purum village, where Model 1 delivers the status 'random' and the more realistic Model 2 delivers the status 'asymmetric' (Table 6). This case also illustrates that whether a configuration of matrimonial relations should be attributed to a symmetric/asymmetric and random/nonrandom process is very sensitive to the way matrimonial relations are defined.

In the three case studies, the methodology presented here showed with great confidence that the observed marriage configurations were not random. Though biases in ethnographic data collection also contribute to the picture, the high confidence scores obtained suggest that the norm was actually operating within these societies. In particular, an old debate about the case of Purum village has been revisited and solved.

Acknowledgements

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Appendix A. Random matrices

Appendix A.1. Random marriage matrix

The $n \times n$ random marriage matrix $\mathbf{W}$ is initialized to the zero matrix. The following operations are performed N times (N the total number of marriages): choose uniformly i in $\{1, \ldots, n\}$; choose uniformly $j \neq i$ in $\{1, \ldots, n\}$; increment entry W_{ij} of $\mathbf{W}$ by 1.

Appendix A.2. Random matrimonial relation matrix

The $n \times n$ random binary matrix $\mathbf{A}$ of matrimonial relations is initialized to the zero matrix. Repeat {choose uniformly i in $\{1, \ldots, n\}$; choose uniformly $j \neq i$ in $\{1, \ldots, n\}$; if $A_{ij} = 0$ then set $A_{ij} = 1$} until matrix $\mathbf{A}$ has exactly k nonzero entries (k the total number of matrimonial relations).

A program for performing the computation of the asymmetry indexes and the corresponding statistical analysis can be downloaded freely (Legendre and Gauzens, 2013–2017): open the data files *Purum_Ackerman.nw0*, *TerkuVandanam.nw0* and *Wailolong.nw0* in the directory *misc/marriage* and use the option **Asym**.

Appendix B. Demographic model

Appendix B.1. Demographic parameters

The demography is described by a female-based age-classified life cycle (Caswell 2001 Caswell (2001)). The time step of the discrete time population dynamics model built on the life cycle is of one year. Males have the same survival rates as females. Survival rates were adapted from mortality tables for females in India (1970-1972) from UN report (United Nations (1982), p. 314), and fecundity rates were adapted from Myanmar data (1951) (Table B.7). The resulting demographic descriptors are given in Table B.8.

Table B.7: Age-specific survival and fecundity rates.

age class	survival rates	fecundity rates
0	0.85	
1	0.95	
2-14	0.99	
15-19	0.99	0.10
20-24	0.99	0.25
25-29	0.99	0.25
30-34	0.99	0.20
35-39	0.99	0.15
40-44	0.99	0.05
45-49	0.98	0.01
50+	0.9	

Table B.8: Demographic descriptors (y = years).

growth rate λ		1.015
life expectancy		35 y
life expectancy after maturity (15 y)		50 y
generation time T		27 y
stable age distribution		
	1-15	42%
	16-30	28%
	31-45	19%
	46-60	9%
	61+	2%

Appendix B.2. Simulation

The program simulates a population of males and females partitioned into classes over a definite time horizon. By construction of an individual-based model, the population trajectories are stochastic (demographic stochasticity). Other stochastic events are included: population regulation, choice of husband...

The program maintains two arrays, one for the females and one for the males. Each entry in these arrays describes the characteristics of an individual that are relevant for the simulation. For example, in each female entry is recorded her date of birth, death, her mother, father, her actual husband, her male and female children...

At initialization, a pool of K individuals is created, where K is the carrying capacity (the maximum number of individuals the environment can sustain). The initial individuals, males and females in equal proportions and of the same age (20 years), are partionned into n classes at random. To these males and females, the marriage procedure is applied every year, and newborns progressively enter the population. The uneven initial population structure creates transient oscillatory dynamics that are stabilized by time $t = 100$.

The main loop of the program performs the following operations from a year to the next:

1. *Survival.* Individuals die or survive according to age-specific survival rates using the Bernoulli distribution.
2. *Reproduction.* Married females give birth according to age-specific fecundity rates using the Poisson distribution. The gender of newborns is drawn according to the Bernoulli distribution with mean 0.5. Children inherit the class of their father.
3. *Marriage.* For each non married female in the population, it is determined whether she can marry, depending on the marriage constraints (demographic and cultural). If this is the case, a list of potential (non married) husbands is established, also based on marriage constraints. If

the list is non empty, a random element is drawn. Then marriage takes place between the selected man and woman. Offspring will be born the next year.

4. *Regulation.* It is assumed that the population cannot overshoot a predefined population ceiling, or carrying capacity K. When the number M of individuals is above K, $M - K$ of them (drawn at random) are set dead. This density dependence procedure reflects altogether the limitation of resources, diseases, etc. When the population is below the ceiling, no kill is performed.
5. *Increase time step.* $t \leftarrow t + 1$.

Appendix B.3. Marriage

1. Demographic constraints are: age at first reproduction, age compatibility between partners.
2. Cultural constraints are: prohibition to marry within own class (exogamy), sex-specific minimum age to marry, not marry again for some time in case of death of one spouse.

See Table B.9.

Table B.9: Marriage parameters (y = years).

woman minimal age to marry	17 y
man minimal age to marry	19 y
maximal age difference between spouses	10 y
minimal time before new marriage	2 y

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