# Supplementary information S1

This section presents the computation of $\mathbb{E}(T(i,j))$, the expected trophic similarity between two species $i$ and $j$ in a random graph.

We denote $P_i$ the set of predators of species $i$, $p_i$ its set of preys. For any set $A$, $|A|$ denotes the number of its elements.

The trophic similarity of species $i$ and $j$ is defined as the number of shared predators $|P_i \cap P_j|$ plus the number of shared preys $|p_i \cap p_j|$ over the total number of predators $|P_i \cup P_j|$ plus the total number of preys $|p_i \cup p_j|$. Hence, the expected trophic similarity is

$$\mathbb{E}(T(i,j)) = \mathbb{E}\left(\frac{|P_i \cap P_j| + |p_i \cap p_j|}{|P_i \cup P_j| + |p_i \cup p_j|}\right).$$

The expectation has the general form

$$\mathbb{E}(T(i,j)) = \sum_k k\mathbb{P}(X = k), \tag{1}$$

where $k$ denotes the possible values of the ratio $\frac{|P_i \cap P_j| + |p_i \cap p_j|}{|P_i \cup P_j| + |p_i \cup p_j|}$.

We first compute the similarity of predators, disregarding preys, and assume that $|P_i| \leqslant |P_j|$. Next we compute the similarity of preys, disregarding predators, under the assumption $|p_i| \leqslant |p_j|$. Finally, we consider the general case.

We note that, if $|P_i \cap P_j| = N$ then

$$|P_i \cup P_j| = |P_i| + |P_j| - |P_i \cap P_j| = |P_j| + |P_i| - N.$$

Let us assume $|P_i| \leqslant |P_j|$. According to (1), the expected value of the predator similarity $PT_{i,j}$ is

$$\mathbb{E}(PT_{i,j}) = \mathbb{E}\left(\frac{|P_i \cap P_j|}{|P_i \cup P_j|}\right) = \sum_{N=0}^{|P_i|}\left(\frac{N}{|P_j| + |P_i| - N}\right)\mathbb{P}(|P_i \cap P_j| = N).$$

To compute $\mathbb{P}(|P_i \cap P_j| = N)$ we think of an urn in which there are $|P_i|$ white balls and $S - |P_i|$ black balls. The probability of finding $N$ white balls when $|P_j|$ balls are sampled from the urn *without replacement* is given by the hypergeometric distribution:

$$\mathbb{P}(|P_i \cap P_j| = N) = \frac{\binom{|P_i|}{N}\binom{S-|P_i|}{|P_j|-N}}{\binom{S}{|P_j|}} = \mathcal{H}(N, S, |P_i|, |P_j|).$$

We obtain

$$\mathbb{E}(PT_{i,j}) = \sum_{N=0}^{|P_i|}\left(\frac{N}{|P_j| + |P_i| - N}\right)\mathcal{H}(N, S, |P_i|, |P_j|). \tag{2}$$

In the same way, for $|p_i| \leqslant |p_j|$, we have the expected prey similarity:

$$\mathbb{E}(pT_{i,j}) = \sum_{n=0}^{|p_i|} \left( \frac{n}{|p_j| + |p_i| - n} \right) \mathcal{H}(n, S, |p_i|, |p_j|). \tag{3}$$

The assumptions $|P_i| \leqslant |P_j|$ (respectively $|p_i| \leqslant |p_j|$) are easily released by replacing $|P_i|$ by $\min(|P_i|, |P_j|)$ and $P_j$ by $\max(|P_i|, |P_j|)$ (respectively $|p_i|$ by $\min(|p_i|, |p_j|)$ and $p_j$ by $\max(|p_i|, |p_j|)$) in (2) and (3):

$$\begin{aligned}
\mathbb{E}(PT_{i,j}) &= \sum_{N=0}^{\min(|P_i|,|P_j|)} \left( \frac{N}{|P_j| + |P_i| - N} \right) \mathcal{H}(N, S, \min(|P_i|, |P_j|), \max(|P_i|, |P_j|)), \\
\mathbb{E}(pT_{i,j}) &= \sum_{n=0}^{\min(|p_i|,|p_j|)} \left( \frac{n}{|p_j| + |p_i| - n} \right) \mathcal{H}(n, S, \min(|p_i|, |p_j|), \max(|p_i|, |p_j|)).
\end{aligned}$$

Finally, we consider simultaneously predators and preys. The probability to obtain $N$ and $n$ is, as these events are independent in a random graph,

$$\mathbb{P}(|P_i \cap P_j| = N) \times \mathbb{P}(|p_i \cap p_j| = n).$$

Considering all possible couples $(N, n)$, we obtain the expectation of the trophic similarity between two species $i$ and $j$:

$$\mathbb{E}\left(T(i,j)\right) = \sum_{N=0}^{\min(|P_i|,|P_j|)} \sum_{n=0}^{\min(|p_i|,|p_j|)} \left( \frac{N+n}{|P_i| + |P_j|) - N + |p_i| + |p_j| - n} \right) \times$$
$$\mathcal{H}(N, S, \min(|P_i|, |P_j|), \max(|P_i|, |P_j|)) \times \mathcal{H}(n, S, \min(|p_i|, |p_j|), \max(|p_i|, |p_j|)).$$