Supplementary Information
Neuronal Network Connectivity and
the Distribution of Times in
Up-States

K. Dao Duc¹ P. Parutto¹ X. Chen² J. Epsztein³ A. Konnerth² D. Holcman¹ *

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In this supplementary information, we show that the oscillation peaks in
the distribution of time in the Up-states is also present in a mean-field model
with inhibition. We further show that there are no oscillation peaks in the
down-state distribution and the decay follows a single exponential.

1 Up-Down state mean-field model with Inhibition

We show here that adding inhibition does not affect the oscillation property
for the time spent in the Up-state. We use the classical mean-field model
of [1], where added an inhibitory component that can also be driven by

*¹Ecole Normale Supérieure, 46 rue d’Ulm 75005 Paris, France. ² Institute of Neuroscience Biedersteiner Str. 29, D-80802 Munchen Germany. ³ Institute of Neurobiology INMED-INSERM U901 Marseille, France
depression, formulated as

\[
\begin{aligned}
\tau \dot{V}_E &= -V_E + J_E \mu_E U R(V_E) - \mu_I U J_{IE} R(V_I) + \sqrt{\tau} \sigma \omega_1 \\
\tau \dot{V}_I &= -V_I - J_I \mu_I U R(V_I) + \mu_E U J_{EI} R(V_E) + \sqrt{\tau} \sigma \omega_2 \\
\dot{\mu}_E &= \frac{1-\mu_E}{\tau_e} - U \mu_E R(V_E) \\
\dot{\mu}_I &= \frac{1-\mu_I}{\tau_e} - U \mu_I R(V_I),
\end{aligned}
\]

(1)

where $V_E, \mu_E$ (resp. $V_I, \mu_I$) are the average voltage (in mV) and depression variables of the excitatory (resp. inhibitory) population. $J_E$ and $J_I$ are the average synaptic strength of the two populations while $J_{EI}$ ($E \rightarrow I$) and $J_{IE}$ ($I \rightarrow E$) represent the average synaptic strength of the connections between the Excitatory and Inhibitory populations. $\omega_1, \omega_2$ are two independent δ-correlated white noines of mean zero and variance one. All other parameters are given in table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$J_E$</th>
<th>$J_I$</th>
<th>$J_{EI}$</th>
<th>$J_{IE}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>0.0156</td>
<td>$2J_E$</td>
<td>0.2$J_E$</td>
<td>$0.2J_E$</td>
</tr>
</tbody>
</table>

Table 1: Parameters for the two-population model

We study the dynamics in three different cases: first, the parameters are given in table 1 (fig. 1A-B), we then study the case where there is no feedback on the inhibition. the parameters are $J_{EE} = 0.0195; J_{IE} = 0.2 * J_{EE}; J_{EI} = 2 * J_{EE}; J_{II} = 0$. Finally we increase the inhibition feedback to $J_{EE} = 0.024; J_{IE} = 0.4 * J_{EE}; J_{EI} = 2 * J_{EE}; J_{II} = J_{EE}$. In all cases, the Up-Down state behavior can be observed as show in in Fig. 1A-F. In the main of the manuscript, we study the excitation only.

2 Distribution of times in the Down-state

Using the mean-field equations with no inhibition (see main text), we obtained statistics for the time in the Down-state ($V \leq 2.37$, which is voltage-position of the saddle point in the phase-space). We find that the distribution
Figure 1: Up-Down state mean-field model with Inhibition. A. Dynamics of the 4 parameters: \( V_E, V_I, \mu_E, \mu_I \). B. Histogram of the time in Up and down states. C-D. Like in A. with \( J_{EE} = 0.0195; J_{IE} = 0.2 \times J_{EE}; J_{EI} = 2 \times J_{EE}; J_{II} = 0 \). E-F. Parameters are \( J_{EE} = 0.024; J_{IE} = 0.4 \times J_{EE}; J_{EI} = 2 \times J_{EE}; J_{II} = J_{EE} \).
<table>
<thead>
<tr>
<th>Parameters</th>
<th>$a$ (95% conf. int.)</th>
<th>$b$ (95% conf. int.)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulations</td>
<td>2188 (2183, 2194)</td>
<td>-2.38 (-2.39, -2.37)</td>
<td>0.99</td>
</tr>
<tr>
<td>Auditory Cortex</td>
<td>30.77 (19.05, 42.5)</td>
<td>-1.82 (-2.76, -0.88)</td>
<td>0.77</td>
</tr>
<tr>
<td>Barrel Cortex</td>
<td>192.9 (163.5, 222.3)</td>
<td>-10.72 (-12.42, -9.02)</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Table 2: Fitted parameter and error for the Down-state duration.

is well approximated by single exponential exponential (Fig. 2a), in agreement with the classical Kramer’s escape problem from an attractor. We further estimated the distribution of times in the Down-state from for experimental data (Fig. 2b-c). We also found that the distribution is well approximated a single exponential, in agreement with the predictions of the model, as shown in fig. 2 and table 2. We fitted the data with a single exponential model $f(t) = ae^{bt}$, using the fitting toolbox of Matlab. For the Auditory and Barrel Cortex recordings, we set downs states as regions for which the membrane potential is less than 6mV above the resting potential. In order to keep the baseline constant we subtracted to the data the moving average with a window of 10s (30s for Barrel cortex data) we then subtracted the minimal value of the data to have only positive values.
Figure 2: Distribution of time in the Down-state.
References